AFRL-AFOSR-VA-TR-2018-0347

Categorical Approach to Agent Interaction

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09/04/2018
Final Report

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTA2
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From - To)*
03/09/2018	Final Report	2013/12/01 -- 2018/11/30

4. TITLE AND SUBTITLE
Categorical approach to agent interaction

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-14-1-0031

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
David I. Spivak

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Massachusetts Institute of Technology
77 Massachusetts Ave.
Cambridge, MA 02139

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
875 North Randolph Street, Room 3112,
Arlington, Virginia 22203

10. SPONSOR/MONITOR'S ACRONYM(S)
AFOSR

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for Public Release; Distribution is Unlimited

13. SUPPLEMENTARY NOTES
N/A

14. ABSTRACT
A full abstract was submitted online; it does not appear to fit in this field. Here is the first paragraph:

During the final year of this grant, my team and I produced a book detailing several applications of category theory; it will be published by Cambridge University Press. In it we gave a mathematical formulation of Censi's theory of collaborative design, as well as discussing resource theories, database integration techniques, and the behavior of interconnected systems of dynamical systems.

15. SUBJECT TERMS
Category Theory, Agent Interaction, Networks, Information, Communication, Learning.

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			David I. Spivak
					19b. TELEPHONE NUMBER *(Include area code)*
U	U	U	UU		510-684-6425

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Final report for AFOSR grant FA9550-14-1-0031
Categorical Approach to Agent Interaction

David I. Spivak

September 3, 2018

This report, submitted to Doug Riecken at the Air Force Office of Scientific Research, summarizes my group's scientific progress on the research sponsored by AFOSR grant FA9550-14-1-0031, titled *Categorical approach to agent interaction*.

Broadly speaking, the scientific questions that inspire my work are:

- What happens—what are the properties of the new entity formed—when multiple agents interact by exchanging resources, physical forces, signals, or information of any sort?
- How can stronger communication between agents be established, so that they can more effectively interact?
- What conceptual tools can we offer scientists to help them better organize, learn from, and communicate their information with others?
- Finally, what is the structure of a learner, and how does learning change the learner?

Category theory is a theory of relationships and composition. As such it has become a gateway to pure math, providing rigorous connections between algebra and number theory, logic and programming, geometry and topology, and probability and measure. My goal during this grant was to show that category theory is also broadly applicable outside of mathematics, from data and knowledge, to information and communication, to dynamics and safety.

Below I will briefly summarize the work that this grant has enabled my team to complete over the past five years.

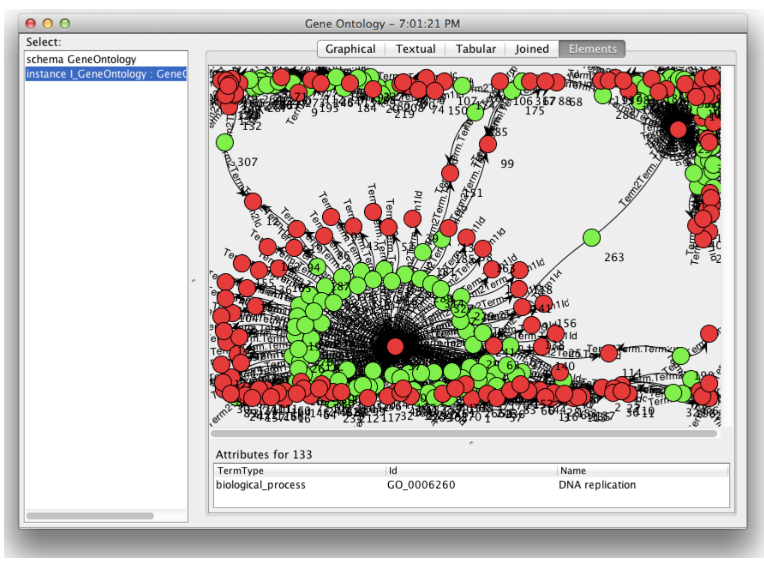
1 Summary of scientific progress

In section 1.1, I will discuss our work on using category theory to facilitate collaboration and communication in engineering. In section 1.2, I will discuss generalizations of Feynman diagrams and their applications to physics, knowledge representation, and belief propagation. In section 1.3, I will discuss compositions of dynamical systems and their behavior, which led to a new method for solving systems of nonlinear equations. Finally, in section 1.4, I will discuss recent work on a compositional approach to supervised learning algorithms.

1.1 Toward interoperability in engineering

To understand what information *is*, we should understand how it is transmitted between agents. Being agents ourselves, we should consider how we transmit information amongst ourselves. But transmitting information is almost never an easy task! The problem arises throughout industry. For example, we see it in "databases that don't talk to each other," in the difficulties that emerge in large scale design projects, and in the lack of effective communication between materials scientists that led to the need for a materials genome project.

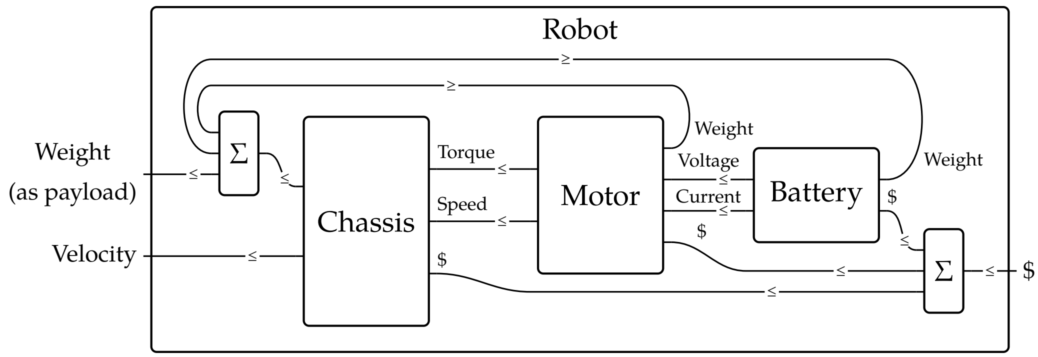
Integrating databases, model management. Each engineer and scientist has his or her own model of the world that is in some sense unique and in some sense shared with a broader community. But this is also the case in formations of autonomous robots: each may have its own model of the world, and in order for the robots to work effectively together they must integrate their information. No matter who is using it—humans or robots—information needs to be structured in order to be usable. Category theory is well-equipped to describe very general sorts of data structures.



Building off previous work on *functorial data migration*, we gave a new description of relational algebra and equational formulations for certain fragments of the categorical database model. This allows it to integrate more with traditional systems, while exhibiting various operations that can be performed category-theoretically, but not in any other known formalism (e.g. relational). These operations were later reduced to practice by the new company *Categorical Informatics*; see below.

We also showed that a large class of constraints (called 'embedded dependencies') are exactly what algebraic topologists call 'lifting problems in the sense of Quillen'. Finally, we worked with researchers from NIST to give a real-world data integration example, for use in additive manufacturing.

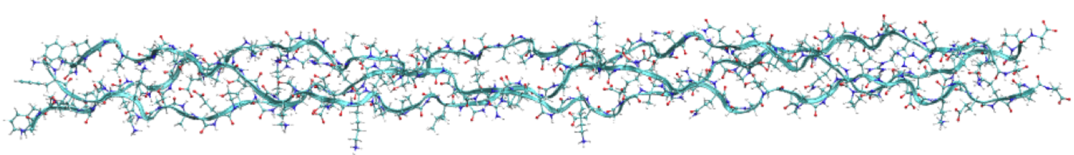
Collaborative design. Designing a solution to an engineering problem requires a collection of teams to work on different aspects of the problem. The teams are somehow both independent from and dependent on each other: each team is tasked with designing certain aspects of the final product, but design decisions of one can affect those of another. In his pioneering work on the mathematics of collaborative design, or co-design, Andrea Censi (then at MIT, now at ETH) provided not only a neat formalism but an open-source programming language for solving such non-convex optimization problems.



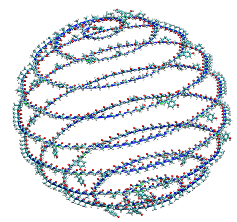
In our new book on applied category theory, *Seven Sketches in Compositionality*, we took Censi's work and showed that it fits beautifully into the theory of profunctors, which form a compact closed category. The well-known string-diagrams for compact closed categories are exactly the sorts of diagrams that Censi uses, and the semantics fits perfectly with Censi's semantics as well.

Materials science. Proteins such as collagen or spider silk are remarkably cheap and energy-efficient to produce (spiders make their silk—stronger pound for pound than steel—simply by eating flies; compare to steel production). The remarkable properties of protein materials arises not from the atoms that compose them, but from the way they are put together. For example, our stretchable, breathable, water-proof skin is a protein material; rather than being made from exotic materials, it is made from a few basic and ubiquitous building blocks. The magic, so to speak, is in the layered structures into which these building blocks are arranged.

Materials scientists want to understand how these layered structures (e.g. amino acids bonded together, wrapped into helices, twisted into coils-of-coils, gathered into arrays) lead to the desired material properties.



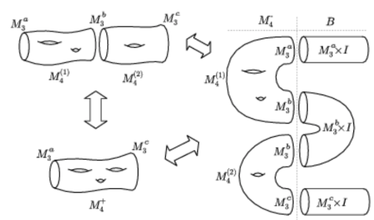
The modern study of materials uses molecular dynamics simulations to study material properties. Together with Professor Markus Buehler (at MIT), we developed an operadic programming language for building complex materials from an existing library of simpler building blocks (each of which is recursively built using the same procedure). This is now an open-source python library.



1.2 2D quantum field theories and cobordisms

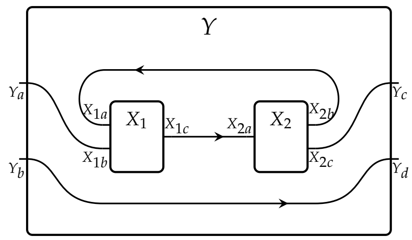
Roughly speaking, topological quantum field theory is the study of how to consistently attach phase spaces to space-time manifolds. We will discuss a new homotopical approach to 2D quantum field theories and a new connection between cobordisms and traced monoidal categories, which are the categorical setting for feedback systems.

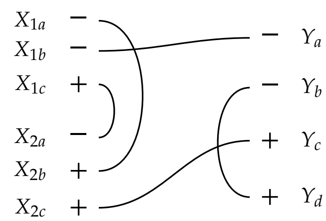
2D quantum field theories. A *cobordism* is an n-dimensional manifold whose boundary has been divided into two disjoint manifolds. Cobordisms form the morphisms of a monoidal category, and a *topological quantum field theory* is a monoidal functor from it to the category of vector spaces.



Thus 2-dimensional TQFTs are representations of the category of 2-dimensional cobordisms. We showed that this category has a simple combinatorial description in terms of 'homotopy cospans'.

Traced categories via cobordisms. 1-dimensional cobordisms are quite simple: just directed wires connected nodes, each labeled by either + or −. And yet we proved that if one considers lax rather than strong representations of 1-cobordisms, the result is Joyal, Street, and Verity's notion of *traced monoidal category*.



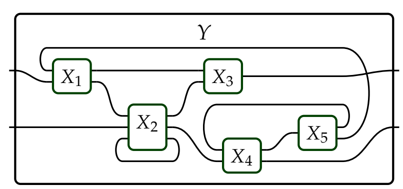


Thus the sorts of categorical structure that come up when systems exhibit feedback are strangely linked with lax topological 1-dimensional quantum field theories. It is still unclear whether this connection is more than just a coincidence.

1.3 Compositionality and dynamics

I am quite interested in dynamical systems, and more generally in anything which could be said to have behavior that occurs in time. In particular, I think it is important to make sense of what emerges when systems of dynamical systems are interconnected, disconnected, or can change their interconnection pattern at will. This work led to a new method for solving systems of nonlinear equations.

Connecting dynamical systems (discrete and continuous). Consider the diagram below, in which machines $X_1, \ldots, X_5$ have been wired together to form a new machine Y:



Each of the machines represents an *open dynamical system*: a formal mathematical entity that acts either continuously or on a discrete-time clock, taking in inputs, changing its internal state, and producing outputs. These outputs then propagate through the wires to other component subsystems. Feedback allows the system to have a notion of control. The whole system Y is another machine of this sort, and can itself be plugged in as a component to some further system of systems. This fractal-like compositionality is captured by an operad.

The goal of the operadic approach is to understand the behavior of the whole given that of the parts and their interaction. We showed in various situations—discrete, continuous, probabilistic, etc.—how to obtain a formula for the behavior of the whole from the behaviors of its parts.

There are a few possible objections to the usefulness of this approach. First, one may not just want a formula for the resulting system, but also to prove properties about it from those of the internal systems. Second, many systems involve both continuous and discrete-time behavior; these are known as hybrid systems, and they are extremely important in robotics. Third, real-world systems do not always maintain a fixed interaction (wiring) pattern. I'll address these below.

Mode-dependent dynamic systems. First, consider systems that do not maintain a fixed wiring pattern. The number of ports of real-world systems can change based on its internal state: for example, a person may close their eyes or a robot may retract a mechanical arm. This leads one to consider what I call the machine's *communicative mode*: I might be in the mode of 'sleep', and the robot may be in the mode of 'make room for another to pass'.

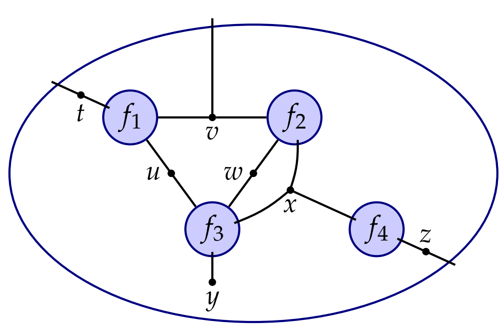
The communicative modes of all the agents in a system also determine the connection pattern between the agents. For example, when two friends are in a certain mode, they connect together and communicate; other times they do not. With my computer, as long as the computer is in 'on' mode, I can communicate with it whenever I'm in the mode (mood) to do so.

I wanted to capture this using an operad, one for systems that change their wiring pattern based on communicative modes. I produced the relevant mathematical formalism, and together with a student we worked out a basic application to the neuroscience of vision.

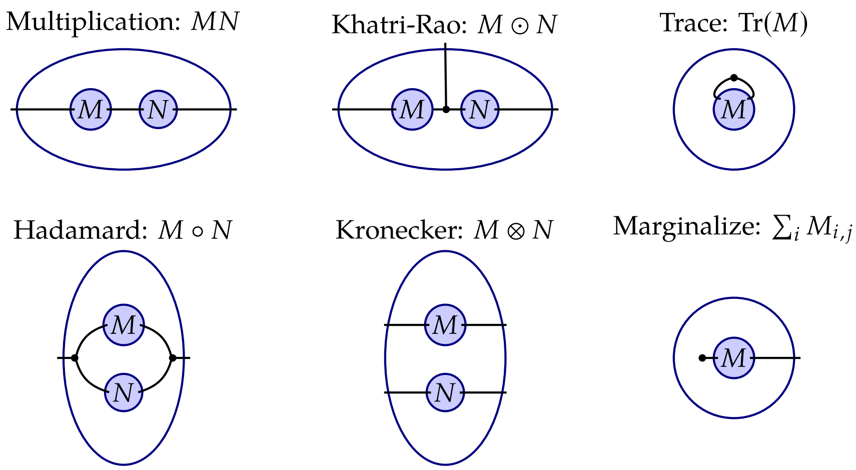
Temporal theory, contracts, behavior. Relating to the first and second objections (wanting not just formulas but properties of interconnected systems, and dealing with both continuous and discrete behavior at the same time), my team developed a temporal type theory—the first of its kind as far as we know—for a very general sort of behavioral system that includes hybrid systems as a special case. With this type theory and its attendant logic, one can prove properties of interconnected components, be they discrete, continuous, deterministic or non-deterministic, delayed, etc.

This work was mainly sponsored by a NASA grant, so I won't go into more details. But since the NASA grant stemmed from my work on the present AFOSR grant, it seems appropriate to mention it here.

Hypergraph categories. Hypergraph categories—so-named because the sorts of composition operations that they axiomatize can be drawn as hypergraphs—have been studied for applications in electrical circuits, database query languages, belief propagation, and free probability, just to name a few.



Roughly speaking, these are settings in which generalized matrix (or tensor) operations can be performed: matrix multiplication, Kronecker product, Khatri-Rao product, marginalization, Hadamard product, and trace.



But they can also capture behavioral relations that change through time, e.g. for the interconnection of dynamical systems (as understood from a behavioral, Willems-style perspective).

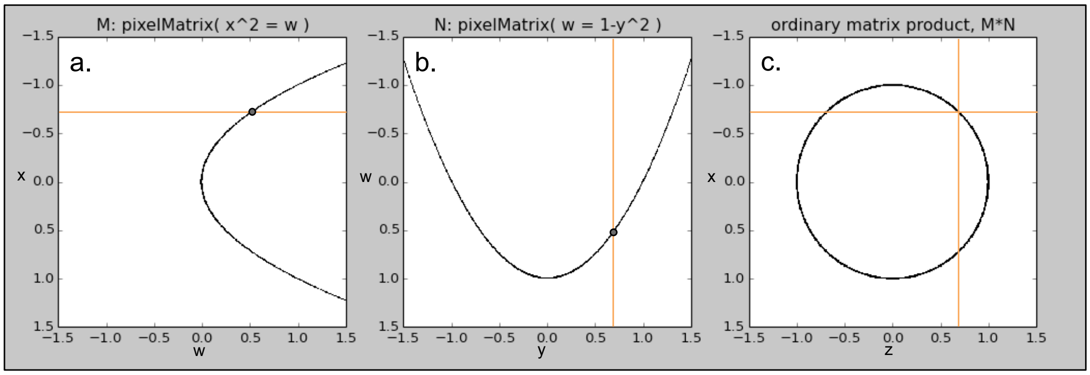
In a recent paper, we recast hypergraph categories using the operad of cospans, proved a new strictification result, and corrected an error that, for some reason, continually reappears in the literature.

Decomposition spaces. While compositionality is a major theme throughout my work, I have also worked on decomposition, whereby one can take a given system and contemplate all possible ways it can be broken up into smaller pieces, in a recursive way. We considered a notion of decomposition space that emerged almost simultaneously in combinatorics and in homological algebra, showing that decomposition spaces form a very interesting sort of category. Namely, the category of decomposition spaces is locally a topos. In particular it is locally Cartesian closed, so it has a good notion of polynomial functor. It will be interesting to see what sorts of combinatorial possibilities this opens up.

Pixel array method. My work on interconnected systems of dynamical systems and their properties, such as how the steady states of the whole are controlled by those of the parts, led to a new method for solving systems of nonlinear equations. Roughly speaking, the steady states of each system are arranged as a n-dimensional tensor, where n is the total number of inputs and outputs of the system. These tensors can then be combined using the tensor arithmetic discussed above, and the result will always be the tensor of steady states for the whole system. But the idea that makes this work is more basic, and works for arbitrary systems of equations.

One takes any equation, plots it as a pixel array (some pixels on, some off), and considers that as a matrix. For example, here we show the plots of two parabolas,

$x^2 = w$ and $w = 1 - y^2$. The matrix product of these two plots is shown to the right; it is no coincidence that it is the plot of their solution, $x^2 = 1 - y^2$, or $x^2 + y^2 = 1$:



I had some students write some software, into which a user can input information like the following:

Solve relations: $\cos\left(\ln(z^2 + 10^{-3}x)\right) - x + 10^{-5}z^{-1} = 0$ (Equation 1)

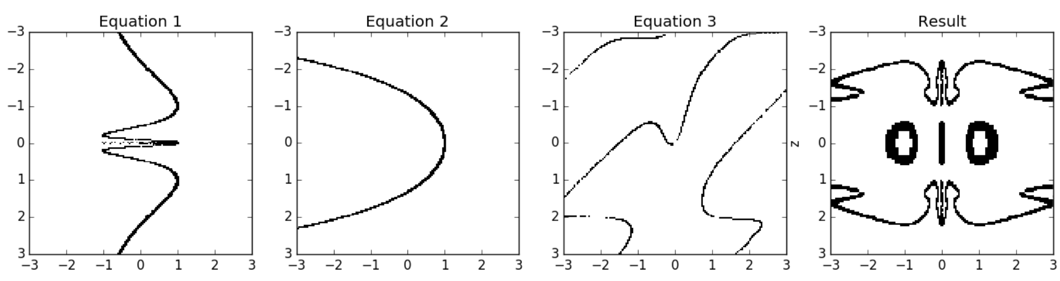
$\cosh(w + 10^{-3}y) + y + 10^{-4}w = 2$ (Equation 2)

$\tan(x + y)(x - 2)^{-1}(x + 3)^{-1}y^{-2} = 1$ (Equation 3)

Discretize by: $w, x, y, z \in [-3, 3]@125$

Expose variables: (w, z)

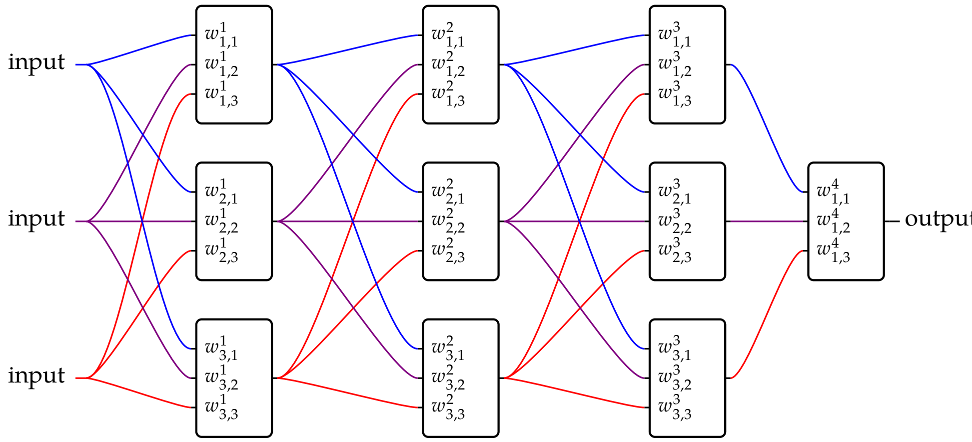
The program will output a plot of all solutions to the resulting system, as shown here:



This work was then sponsored by another AFOSR grant, under Program Officers Fred Leve and Jean-Luc Cambier, but again, it stemmed from this AFOSR grant.

1.4 Supervised machine learning

Backprop as a functor. The final project I'll discuss is some work we did on supervised learning algorithms, e.g. those used in deep learning. We showed that neural networks and learning algorithms each form a monoidal category, and that the gradient descent, back propagation algorithm constitutes a monoidal functor between them. This means that supervised learning algorithms are compositional.



We propose to carry this work further in a next incarnation of this grant. One direction of interest is a rigorous category-theoretic analogy we found between the monoidal category for learning algorithms and that for what are called "open economic games," due to some researchers at Oxford and elsewhere. We believe there may be room for a combination of economic game-playing agents and learning algorithms like those used in deep neural networks.

2 Transitions: companies and new grants

The work from this grant led in many exciting new directions, both academic and beyond. The mathematical frameworks and algorithms are being implemented in working code by several companies, and the ideas spun out two new grants.

2.1 Companies using this work

Categorical informatics. I am a co-founder and chief mathematician at Categorical Informatics Inc, which does database migration and integration. Research has shown that over half of data integration projects fail, a major loss for companies. In fact, about 40% of all IT budgets are spent on data integration! With any data migration software, a project may take the program more than a day to complete. However, with other software, the process can fail at the very end, in that it does not conform to the target schema. With our category-theoretic technique, the program can tell the user at compile time, before the work is actually done, whether or not it will succeed; if the program says it will succeed, it is guaranteed to do so.

This work has been funded by grants from NIST and the NSF. We will be looking for venture capital investment this fall.

Statebox. I am formally an advisor at Statebox, a company that uses category theory to make a highly robust and flexible open-source programming language. The relation to this grant is the use of operads and their algebras for interconnecting simpler programs to form more complex ones.

Airbus. Researchers at airbus have reached out to me, to let me know that they are using my work in their company. These groups are interested in model management and knowledge representation, as well as systems of systems. Airbus has a small group dedicated completely to applications of category theory.

2.2 New grants

AFOSR. Work showing that steady states of interconnected (nonlinear) dynamical systems can be calculated by matrix arithmetic led to a breakthrough, namely the pixel array method discussed above. This in turn led to my seeking and receiving a new AFOSR grant to fund research into it and other compositional methods for analyzing systems of systems.

NASA. Work on interconnected dynamical systems also led to a grant with NASA, where we wanted to prove properties of the National Airspace System using properties of various components, such as airplanes, pilots, air traffic controllers, radars, collision avoidance systems, etc.

3 Papers, presentations, and collaborations

Below I will list some publications and presentations with which I have been involved, during the performance period of this AFOSR grant. Some are about the above topics, others are about tangential 'spin-off' topics.

3.1 Books

- Fong, B.; **Spivak, D.I.** (2018) *An Invitation to Applied Category Theory: Seven Sketches in Compositionality*. Cambridge University Press (*to appear*).
- Schultz, P.; **Spivak, D.I.** (2017) *Temporal Type Theory: A Topos-theoretic Approach to Systems and Behavior.* Springer Birkhäuser (*to appear*).
- **Spivak, D.I.** (2014) *Category Theory for the Sciences*. Cambridge: MIT Press.

3.2 Journal papers, book chapters, conferences, and preprints

- Kock, J.; **Spivak, D.I.** (2018) "Decomposition space slices are toposes". *Submitted.* Available online: https://arxiv.org/abs/1807.06000.
- Fong, B.; **Spivak, D.I.** (2018) "Hypergraph categories". *Submitted.* Available online: https://arxiv.org/abs/1806.08304.

- Liu, C.T.; **Spivak, D.I.** (2018) "Evaluating the Pixel Array Method as Applied to Partial Differential Equations". Available online: https://arxiv.org/abs/1808.01724.
- Speranzon, A.; **Spivak, D.I.**; Varadarajan, S. (2018) "Abstraction, Composition and Contracts: A Sheaf Theoretic Approach". Available online: http://arxiv.org/abs/1802.03080.
- Fong, B.; **Spivak, D.I.**; Tuyéras, R. (2017) "Backprop as Functor: A compositional perspective on supervised learning". Available online: http://arxiv.org/abs/1711.10455.
- **Spivak, D.I.** (2017) "Categories as mathematical models." *Categories for the Working Philosopher*. Oxford University Press. Available online http://arxiv.org/abs/1409.6067
- Wisnesky, R.; Breiner, S.; Jones, A.; **Spivak, D.I.**; Subrahmanian, E. (2017) "Using category theory to facilitate multiple manufacturing service database integration." *ASME. Journal of Computing and Information Science in Engineering* Volume 17, Number 2.
- **Spivak, D.I.**; Schultz, P.; Rupel, D. (2017) "String diagrams for traced and compact categories are oriented 1-cobordisms." *Journal of Pure and Applied Algebra*. Volume 221, Issue 8, pp. 2064 – 2110. Available online: http://arxiv.org/abs/1508.01069
- Wisnesky, R.; **Spivak, D.I.**; Schultz, P. (2017) "Algebraic model management." *WADT'16 post-proceedings*, Springer.
- Schultz, P.; **Spivak, D.I.**; Vasilakopoulou, C.; Wisnesky, R. (2017) "Algebraic databases." *Theory and Applications of Categories*, Volume 32, pp. 547 – 619. Available online: http://www.tac.mta.ca/tac/volumes/32/16/32-16.pdf
- **Spivak, D.I.**; Tan, J.Z. (2016) "Nesting of dynamic systems and mode-dependent networks." *Journal of Complex Networks.* Available online: https://arxiv.org/abs/1502.07380.
- Forssell, H.; Gylterud, H.K.; **Spivak, D.I.** (2016) "Type theoretical databases." *Logical Foundations of Computer Science*. Available online http://arxiv.org/abs/1406.6268.
- **Spivak, D.I.**; Ernadote, D.; Hammami, O. (2016) "Pixel matrices: An elementary technique for solving nonlinear systems." *2016 IEEE International Symposium on Systems Engineering (ISSE)*. Available online http://arxiv.org/abs/1605.00190.
- Kock, J.; **Spivak, D.I.** (2016) "Homotopy composition of cospans." *Communications in Contemporary Mathematics*. Available online: http://arxiv.org/abs/1602.1493699
- **Spivak, D.I.**; Vasilakopoulou C.; Schultz, P. (2016) "Dynamical systems and sheaves." *Submitted*. Available online: http://arxiv.org/abs/1609.08086.
- **Spivak, D.I.**; Dobson, M.R.C.; Kumari, S.; Wu, L. (2016) "Pixel Arrays: A fast and elementary method for solving nonlinear systems." *Submitted.* Available online: https://arxiv.org/abs/1609.00061.

- Lerman, E.; **Spivak, D.I.** (2016) "An algebra of open continuous time dynamical systems and networks." Available online: http://arxiv.org/abs/1602.01017.
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3.3 Invited Presentations

Over the course of this performance period, I spoke in the following seminars and conferences:

- International Category Theory Conference, 2018/07/08

- Topology Seminar, Universitat Autònoma de Barcelona, 2018/07/06
- Toposes in Como, 2018/06/27
- Special Seminar, Max Planck Institute for Mathematics in the Sciences, 2018/05/07
- Applied Category Theory workshop, Lorentz Center, 2018/05/02
- Applied Category Theory workshop, National Institute of Standards and Technology, 2018/03/16
- Oxford Advanced Seminar on Informatic Structures, Oxford 2018/02/01
- Applied topology seminar, University of Aberdeen, 2018/01/30
- Applied Algebra and Topology seminar, MIT, 2017/12/05
- Biomolecular Feedback Systems seminar, MIT, 2017/11/15
- AMBER lab Robotics seminar, Caltech University, 2017/11/03
- Category theory and Logic seminar, McGill University, 2017/10/03
- Applied math seminar, McGill University, 2017/10/02
- Topology Seminar, Universitat Autònoma de Barcelona, 2017/06/12
- Emerging Topics in Network Dynamical Systems workshop, Lorentz Center, 2017/06/08
- Compositionality workshop, Simons Institute for the Theory of Computing, 2016/12/06
- Agenda des Écoles, Grand École Télécom Bretagne, 2016/11/10
- Numerical Methods for PDEs Seminar, MIT Mathematics, 2016/10/26
- E3 Science Lecture Series, Sandia National Labs, 2016/09/27
- Workshop on Probability, Uncertainty, and Decisions, AFRL Dayton, 2016/06/27
- 5th Mini-Symposium on Computational Topology, Boston, 2016/06/15
- Mathematics Colloquium, New York University, Binghamton, 2016/04/07
- Generalized Network Structures and Dynamics workshop, Mathematical Biosciences Institute, 2016/03/22
- Invited Presentation, Amgen, 2016/03/17
- Mathematics Colloquium, University of Massachusetts, Boston, 2015/10/14
- Computational Category Theory Workshop, National Institute of Standards and Technology, 2015/09/28
- Seminar, University of Oslo Department of Informatics, 2015/09/21
- Lunch Seminar, MIT Laboratory for Information and Decision Systems, 2015/06/26
- Invited Presentation, National Institute of Standards and Technology, 2015/06/18
- Invited Presentation, National Institute of Standards and Technology, 2015/06/16
- Foundational Methods in Computer Science conference, Colgate University, 2015/06/06
- Categorical Foundations of Network Theory workshop, Institute for Scientific Interchange, 2015/05/28
- Complex Systems Seminar, University of Pennsylvania, 2015/04/03
- Applied Algebra and Network Theory Seminar, Pennsylvania State University, 2015/03/18
- International Workshop on Design Theory, MINES ParisTech, 2015/01/26
- Programming Languages Seminar, MIT Computer Science and Artificial Intel-

ligence Laboratory, 2014/04/15

- Bar talk, Institute for Advanced Study, 2014/03/20
- Invited Presentation, PARC, 2014/03/03
- Invited Presentation, Amgen, 2014/03/04
- Invited Presentation, Oracle, 2014/02/28
- Topology Seminar, University of Illinois Urbana-Champaign, 2014/02/25
- Programming Languages Seminar, Harvard University, 2014/02/19
- Principles of Programming Seminar, Carnegie Mellon University, 2014/01/23